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A Side-sensitive Modified Group Runs Control Chart with Auxiliary Information to Detect Process Mean Shifts

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ABSTRACT

This study aims to develop a side-sensitive modified group runs control chart using auxiliary information (SSMGR-AI) to enhance the speed of detecting mean shifts in a process. The average run length (ARL) and expected average run length (EARL) criteria are adopted as performance measures of the proposed chart. The performance of the proposed chart is compared to the exponentially weighted moving average chart with AI (EWMA-AI) and the run sum chart with AI (RS-AI), in terms of the ARL and EARL criteria. The results reveal that the optimal SSMGR-AI chart generally outperforms all charts under comparison for detecting shifts in the process mean. An application with numerical data is presented to elaborate the implementation of the SSMGR-AI chart.

Keywords: Auxiliary information (AI), average run length (ARL), expected average run length (EARL), side-sensitivity, side-sensitive modified group runs (SSMGR)

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INTRODUCTION

Control charts are a well-known process used to maintain the quality of production in modern industries and manufacturing sectors. The main purpose of the control chart is to monitor infrequent changes in manufacturing and industrial processes. In 1924, Shewhart first developed the Shewhart $\bar{X}$ control chart which depended on a single characteristic to monitor the process mean

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of product quality (Montgomery, 2009). Later, enormous research on control charts has resulted in different dimensions of the basic chart used to detect dissimilarities in production parameters such as mean, variation or both. The concept of rational sub-grouping is employed to detect unusual patterns in the production process. Efficient estimators of the desired parameters are computed from each sub-group and integrated into the Shewhart, EWMA or CUSUM control chart to detect shifts in the mean, variance or both. Bourke (1991) suggested that the conforming run length (CRL) chart could be used to detect shifts in the fraction of non-conforming items in a production process. Wu and Spedding (2000) developed a synthetic chart which combined Shewhart $\overline{X}$ chart and CRL chart to monitor shifts in the process mean. To enhance the performance of synthetic chart, Gadre and Rattihalli (2004) proposed group runs (GR) control chart to detect shifts in the process mean. By modifying the GR chart, Gadre and Rattihalli (2006) further presented a modified group runs (MGR) control chart which was useful to identify increases in the fraction of non-conforming items and detect shifts in the process mean.

Davis and Woodall (2002) highlighted that side-sensitivity feature could be used to improve the chart performance and proposed the side-sensitive synthetic chart. A side-sensitive group runs (SSGR) chart investigated by Gadre and Rattihalli (2007) had surpassed the Shewhart, synthetic and GR charts. Furthermore, Gadre et al. (2010) proposed a side-sensitive modified group runs (SSMGR) chart, where it was shown that this chart performed better than the Shewhart, synthetic, GR, side-sensitive group runs (SSGR) and MGR charts. To extend the work of Garde et al. (2010), Saha et al. (2018a) proposed the SSMGR double sampling (SSMGRDS) chart and the latter was found to outperform its basic counterparts.

In the last decade, the use of auxiliary information in Statistical Process Control has gained the attention of researchers. If a variable is known for every unit of the population but it is not a variable of interest, then the said variable can be used as an auxiliary variable, where information from the auxiliary variable, called auxiliary information, along with the main variable of interest, enhances the level of precision of the control charting statistic. Riaz (2008) proposed a new Shewhart type chart to monitor the process mean with auxiliary information along with a regression estimator. Riaz et al. (2013) highlighted that in the presence of auxiliary information, a control chart could perform efficiently under normality and non-normality assumptions with estimation effects. Abbas et al. (2014) showed that the exponentially-weighted moving average chart with single auxiliary information (EWMA-AI) performed better than its univariate and bivariate competitors to detect small and moderate shifts. Abbasi and Riaz (2016) found that the use of auxiliary information in-control charts enabled the charts to detect shifts more efficiently and quickly than the charts without auxiliary information.

Haq and Khoo (2016) showed that a new synthetic control chart based on both study and auxiliary variable had performed more effectively than the classical synthetic chart and also its univariate and bivariate competitors. Saha et al. (2018b) developed a variable sample size and sampling interval (VSSI) control chart using auxiliary information (VSSI AI) to monitor the process mean, while the average time to signal (ATS) and expected time to signal (EATS) criteria were adopted as performance measures. A run sum chart for the mean based on auxiliary characteristics (RS-AI) was proposed by Ng et al. (2018) and compared with the Shewhart AI, synthetic AI, and EWMA- AI charts.

In this research, the auxiliary information procedure is incorporated into the existing SSMGR chart, in order to propose the side-sensitive modified group runs chart with auxiliary information (SSMGR-AI) to detect process shifts. The ARL and EARL criteria are applied to measure how quickly the proposed chart can detect infrequent changes in production. An optimal design is conducted to compute optimal parameters of the SSMGR-AI chart by minimizing the out-of-control ARL and EARL values for different mean shifts and shift intervals, respectively. After that, the SSMGR-AI chart is compared with the EWMA-AI and RS-AI charts. Finally, a numerical example, based on generated data, is given to explain the implementation of the proposed SSMGR-AI chart.

Existing Control Charting Method: SSMGR Chart

Gadre et al. (2010) assimilated both the $\bar{X}$ sub-chart and the CRL sub-chart into the SSMGR chart. The upper control limit ($UCL_{\bar{X}}$) and lower control limit ($LCL_{\bar{X}}$) of the $\bar{X}$ sub-chart is calculated as

$$UCL_{\bar{X}} = \mu_0 + k_{\bar{X}} \frac{\sigma_0}{\sqrt{n}} \qquad [1]$$

$$LCL_{\bar{X}} = \mu_0 - k_{\bar{X}} \frac{\sigma_0}{\sqrt{n}}\ , \qquad [2]$$

where, $k_{\bar{X}}$ is the width constant controlling the width of the $\bar{X}$ sub-chart to satisfy the desire in-control performance. When the sample mean $\bar{X}$ falls within $(LCL_{\bar{X}}, UCL_{\bar{X}})$, the sample is called conforming; otherwise, the sample is considered as non-conforming. The conforming run length (CRL) is defined as the number of conforming samples inspected between the $(q-1)^{th}$ and q^{th} non-conforming samples, including the q^{th} non-conforming sample. The value of CRL_q at the q^{th} non-conforming sample is defined as Y_q throughout the paper. The procedure of the SSMGR $\bar{X}$ chart is described by the following steps:

Step 1. Draw n successive products and compute sample mean $\bar{X}$ from a process of following $N(\mu_1, \sigma^2)$ distribution with the in-control mean μ_0 and standard deviation σ. Here $\mu_1 = \mu_0 \pm \delta\sigma$, and when the process is in-control, $\delta = 0$; otherwise, the process is out-of-control.

Step 2. When $\bar{X}$ falls between the limits $UCL_{\bar{X}}$ and $LCL_{\bar{X}}$, the sample is declared as conforming, otherwise, it is considered as non-conforming.

Step 3. If a sample is conforming, return to Step 1. Otherwise, compute Y_q , for q = 1, 2, ….

Step 4. If $Y_1 \leq W_2$ or for $q > 1$, $Y_q \leq W_1$, and $Y_{q+1} \leq W_2$, declare the process as out-of-control, where the q^{th} and $(q+1)^{th}$ non-conforming sample means lie on the same side of the target value μ_0 , while W_l (l = 1, 2) is the lower limit of the CRL sub-chart. Otherwise, return to Step 1.

New Control Charting Method: SSMGR-AI Chart

In this research, information from the primary variable (S) and auxiliary variable (M) are considered to develop the SSMGR-AI chart by incorporating the SSMGR charting approach. The chart statistic is designed in such a way that it detects only the shifts in the process mean of primary variable S.

Assume a joint distribution between the two bivariate normal variates (S, M) with parameters μ_S, μ_M, σ_S^2, σ_M^2 and ρ . Here, μ_S and σ_S^2 indicate the population mean and variance of the primary variable S, while μ_M and σ_M^2 are the population mean and variance of the auxiliary variable M. ρ is the correlation coefficient between S and M. The joint distribution of (S, M) can be expressed as

$$(S,M) \sim N_2\left(\mu_{S0} + \delta\sigma_S, \mu_M, \sigma_S^2, \sigma_M^2, \rho\right), \qquad [3]$$

where $\mu_S = \mu_{S0} + \delta\sigma_S$. Here, δ is the size of the standardized mean shift of variable S. Let $\left(S_{ij}, M_{ij}\right)$ for j = 1, 2……n , denote the i^{th} random sample from a bivariate normal distribution. According to Riaz (2008), an unbiased estimator of μ_{S_i} is given as

$$\hat{\mu}_{S_i}^* = \hat{\mu}_{S_i} + \beta\left(\mu_M - \hat{\mu}_{M_i}\right), \qquad [4]$$

where $\hat{\mu}_{S_i}$ and $\hat{\mu}_{M_i}$ are the i^{th} sample means of S and M, respectively. Here, $\hat{\mu}_{S_i} = \bar{Y}_i = \sum_{j=1}^{n} Y_j / n$, $\hat{\mu}_{M_i} = \bar{M}_i = \sum_{j=1}^{n} M_j / n$, and $\beta = \rho\left(\sigma_S / \sigma_M\right)$. It is noted that $\hat{\mu}_{S_i}$ is a special case of $\hat{\mu}_{S_i}^*$ when $\rho = 0$ (see Equation [4]).

(S, M) follows the bivariate normal distribution and the random variable $\hat{\mu}_{S_i}^*$ follows a normal distribution and can be defined as (Riaz, 2008):

$$\hat{\mu}_{S_i}^* \sim N\left(\mu_{S0} + \delta\sigma_S, \left(\sigma_S^2/n\right)\left(1-\rho^2\right)\right). \qquad [5]$$

Here, $\hat{\mu}_{S_i}^*$ (for i = 1, 2, …) are the quality characteristics monitored by a process. If the process is in-control, the target mean of $\hat{\mu}_{S_i}^*$ is μ_{S0}.

The control limits of the SSMGR-AI chart based on $\hat{\mu}_{S_i}^*$ are obtained by

$$UCL = \mu_{S0} + k\frac{\sigma_S}{\sqrt{n}}\sqrt{1-\rho^2} \qquad [6]$$

$$LCL = \mu_{S0} - k\frac{\sigma_S}{\sqrt{n}}\sqrt{1-\rho^2} \qquad [7]$$

where, k is the control limit coefficient of the SSMGR-AI chart which depends on the desired in-control performance.

The implementation procedures for the SSMGR-AI chart is the same as that of the SSMGR $\bar{X}$ chart of Gadre et al. (2010) which were discussed in above Section. The probability of a non-conforming sample when the process mean has shifted by δ standard deviation is defined as

$$P(\delta) = 1 - \Phi\left(k - \delta\sqrt{n/\left(1-\rho^2\right)}\right) + \Phi\left(-k - \delta\sqrt{n/\left(1-\rho^2\right)}\right).$$

The ARL for the SSMGR-AI chart is formulated as

$$\text{ARL}(\delta) = \frac{1}{P(\delta)} \times \frac{1 + C_1 - C_2 - 2C_1C_2\alpha(1-\alpha)}{C_1C_2\left\{1-2\alpha(1-\alpha)\right\}}, \qquad [8]$$

where $C_l = 1 - \left(1 - P(\delta)\right)^{W_l}$, for l = 1, 2, and

$$\alpha = \frac{1 - \Phi\left(k - \delta\sqrt{n/\left(1-\rho^2\right)}\right)}{P(\delta)}. \qquad [9]$$

Here, α is the probability of the non-conforming sample having an upward shift with a shift size of δ, and $\Phi(\cdot)$ represents the cumulative distribution function of a standard normal random variable.

The expected average run length (EARL) to signal which considers an overall range of shifts (δ_{min}, δ_{max}) is computed as

$$\text{EARL} = \int_{\delta_{\min}}^{\delta_{\max}} \text{ARL}(\delta) f(\delta)\, d\delta, \qquad [10]$$

where $\text{ARL}(\delta)$ is the value of ARL in Equation [8] for the shift δ and $f(\delta)$ are the probability density function (pdf) of the shift δ. The probability that a mean shift will occur in the range $\delta_{\min} \leq \delta \leq \delta_{\max}$ is considered equal. Here, δ is assumed to be uniformly distributed (Sparks, 2000), i.e. $\delta \sim U(\delta_{min}, \delta_{max})$. It can be written from Equation [9] as follows

$$\text{EARL}(\delta_{\min}, \delta_{\max}) = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \text{ARL}(\delta)\, \mathrm{d}\delta. \qquad [11]$$

Optimization Method of the SSMGR-AI Chart

The purpose of an optimal design is to compute the optimal parameters (k, W_1, W_2) in such a way that minimizes the ARL(δ) or ARL ($\delta_{\min}$, $\delta_{\max}$) criterion, for a given ρ and an exact shift size δ or shift interval ($\delta_{\min}$, $\delta_{\max}$). The algorithm to compute the optimal parameters of the SSMGR-AI chart in minimizing ARL *(δ)* consists of the following eight steps:

Step 1. Specify the in-control ARL (ARL_0), sample size (n) and shift size (δ). Then let $W_1 = 0$ and $W_2 = 0$. Additionally, initialize $ARL_{min} = \infty$ and $ARL_{opt} = \infty$. Here, ARL_0 is the target value of ARL (δ) when $\delta = 0$.

Step 2. Start with $W_1 = W_1 + 1$.

Step 3. Set $W_2 = W_2 + 1$.

Step 4. Determine the value of k by solving Equation [8], so that ARL(0) = ARL_0.

Step 5. Calculate *ARL* (δ), for the shift size δ, using Equation [8] and the current values of k, W_1 and W_2.

Step 6. If ARL(δ) < ARL_{min}, then let ARL_{min} = ARL(δ) and return to Step 3. Otherwise, proceed to the next step.

Step 7. If $ARL_{min} < ARL_{opt}$ then $ARL_{opt} = ARL_{min}$. Reset $W_2 = 0$ and return to Step 2. Otherwise, proceed to the next step.

Step 8. ARL_{opt} is recorded as the minimum ARL(δ) value and the corresponding k, W_1 and W_2, values are taken as the optimal parameters of the SSMGR-AI chart which satisfies ARL(0) = ARL_0.

This eight steps algorithm is also presented in a flowchart in Figure 1, in order to facilitate a better understanding of the aforementioned algorithm.

By following the same algorithm as in Steps 1 – 8, the optimal parameters of the SSMGR-AI chart are also obtained by minimising the EARL *(*$\delta_{\min}$, $\delta_{\max}$) value for the shift interval ($\delta_{\min}$, $\delta_{\max}$). The only differences are

(i) the shift interval ($\delta_{\min}$, $\delta_{\max}$) is used instead of an exact shift size δ, and

(ii) Equation [11] is used to compute EARL ($\delta_{\min}$, $\delta_{\max}$), instead of using Equation [8] for computing ARL (δ).

The proposed chart is designed to minimising EARL ($\delta_{\min}$, $\delta_{\max}$) in such a way that the EARL(0) = ARL_0. Note that, when $\rho = 0$ the SSMGR-AI chart is like the basic SSMGR chart.

Using the steps from 1 to 8, an optimization MATLAB program is written to compute the optimal parameters, as well as the minimum ARL (δ) and EARL ($\delta_{\min}$, $\delta_{\max}$) values. Tables 1 to 2 and Tables 3 to 4 report the optimal chart parameters (k, W_1, W_2), for different $n \in \{5,7\}$, $\delta \in$ (0.1, 0.3, 0.5, 0.7, 1, 1.5, 2) and $\rho \in$ (0, 0.25, 0.5, 0.75, 0.95) that minimize the ARL(δ) and EARL ($\delta_{\min}$, $\delta_{\max}$) values, respectively of the SSMGR-AI chart. For example, when SSMGR-AI chart is optimally set to minimize ARL (0.7), i.e. $\delta = 0.7$, $\rho = 0.5$ and $n = 5$ are specified, the value of the optimal parameters are (k, W_1, W_2) $\in$ (1.5694, 1, 5) (see Table 1). Similarly, to minimize the EARL (δ_{min} , δ_{max}) values, the values of the parameters are (k, W_1, W_2) $\in$ (2.0690, 1, 56) for the optimal SSMGR-AI chart, when ($\delta_{\min}$, $\delta_{\max}$) $\in$ (0.5, 1.0), $\rho = 0.25$, $n = 7$ and in-control $ARL_0 = 200$ are specified (see Table 3).

RESULTS AND PERFORMANCE EVALUATION

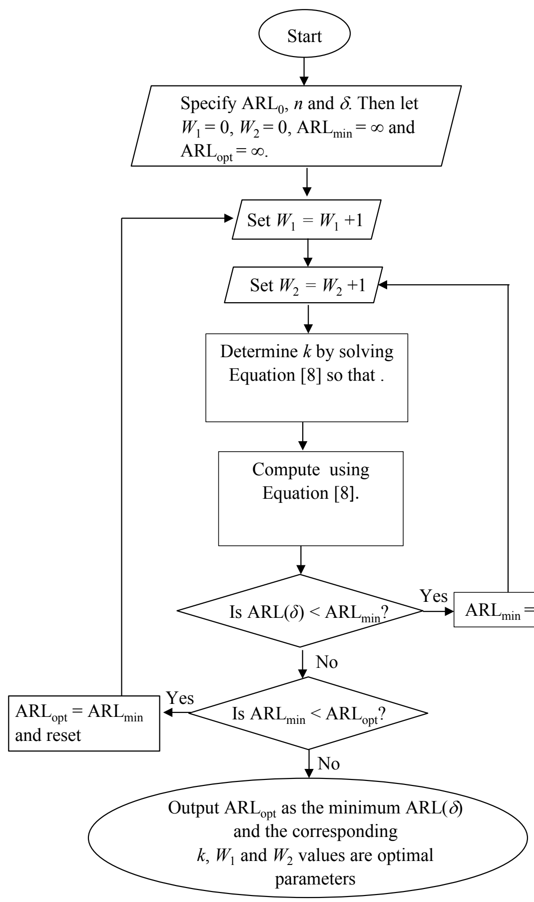

Figure 1. A flowchart showing the algorithm in computing the optimal parameters of the SSMGR-AI chart in minimizing ARL (δ)

Table 1

Optimal parameters of the SSMGR-AI chart for minimizing the ARL (δ) when ARL_0= 200

δ	ρ	$n = 5$			$n = 7$		
		k	W_1	W_2	k	W_1	W_2
0.1	0	2.2447	1	123	2.2447	1	123
	0.25	2.2447	1	123	2.2447	1	123
	0.5	2.2447	1	123	2.2447	1	123
	0.75	2.2315	1	116	2.1888	1	96
	0.95	1.9548	1	33	1.8786	1	23

Table 1 *(Continued)*

δ	ρ	$n = 5$			$n = 7$		
		k	W_1	W_2	k	W_1	W_2
0.3	0	1.9849	1	38	1.9122	1	27
	0.25	1.9733	1	36	1.8875	1	24
	0.5	1.9199	1	28	1.8390	1	19
	0.75	1.7904	1	15	1.7273	1	11
	0.95	1.4674	1	3	1.4674	1	3
0.5	0	1.7449	1	12	1.6868	1	9
	0.25	1.7273	1	11	1.6632	1	8
	0.5	1.6868	1	9	1.6057	1	6
	0.75	1.5694	1	5	1.5249	1	4
	0.95	1.3857	1	2	1.3857	1	2
0.7	0	1.6057	1	6	1.5249	1	4
	0.25	1.6057	1	6	1.5249	1	4
	0.5	1.5694	1	5	1.4674	1	3
	0.75	1.4674	1	3	1.3857	1	2
	0.95	1.3857	1	2	1.3857	1	2
1.0	0	1.4674	1	3	1.4674	1	3
	0.25	1.4674	1	3	1.3857	1	2
	0.5	1.4674	1	3	1.3857	1	2
	0.75	1.3857	1	2	1.3857	1	2
	0.95	1.3857	1	2	1.3857	1	2
1.5	0	1.3857	1	2	1.3857	1	2
	0.25	1.3857	1	2	1.3857	1	2
	0.5	1.3857	1	2	1.3857	1	2
	0.75	1.3857	1	2	1.3857	1	2
	0.95	1.3857	1	2	1.3857	1	2
2.0	0	1.3857	1	2	1.3857	1	2
	0.25	1.3857	1	2	1.3857	1	2
	0.5	1.3857	1	2	1.3857	1	2
	0.75	1.3857	1	2	1.3857	1	2
	0.95	1.3857	1	2	1.3857	1	2

Table 2
Optimal parameters of the SSMGR-AI chart for minimizing the ARL (δ) when $ARL_0 = 370$

δ	ρ	$n = 5$			$n = 7$		
		k	W_1	W_2	k	W_1	W_2
0.1	0	2.3990	1	195	2.3990	1	195
	0.25	2.3990	1	195	2.3990	1	195
	0.5	2.3990	1	195	2.3990	1	195
	0.75	2.3661	1	168	2.3354	1	146
	0.95	2.0866	1	45	2.0051	1	30
0.3	0	2.1201	1	53	2.3990	1	195
	0.25	2.1081	1	50	2.3990	1	195
	0.5	2.0470	1	37	2.3990	1	195
	0.75	1.9256	1	20	2.3354	1	146
	0.95	1.6217	1	4	2.0051	1	30
0.5	0	1.8702	1	15	2.0358	1	35
	0.25	1.8570	1	14	2.0241	1	33
	0.5	1.8112	1	11	1.9692	1	25
	0.75	1.6975	1	6	1.8429	1	13
	0.95	1.4913	1	2	1.5678	1	3
0.7	0	1.7263	1	7	1.6635	1	5
	0.25	1.6635	1	5	1.6217	1	4
	0.5	1.5678	1	3	1.5678	1	3
	0.75	1.4913	1	2	1.4913	1	2
	0.95	1.4913	1	2	1.4913	1	2
1.0	0	1.6217	1	4	1.5678	1	3
	0.25	1.5678	1	3	1.5678	1	3
	0.5	1.5678	1	3	1.4913	1	2
	0.75	1.4913	1	2	1.4913	1	2
	0.95	1.4913	1	2	1.4913	1	2
1.5	0	1.4913	1	2	1.4913	1	2
	0.25	1.4913	1	2	1.4913	1	2
	0.5	1.4913	1	2	1.4913	1	2
	0.75	1.4913	1	2	1.4913	1	2
	0.95	1.4913	1	2	1.4913	1	2

Table 2 *(Continued)*

δ	ρ	$n = 5$			$n = 7$		
		k	W_1	W_2	k	W_1	W_2
2.0	0	1.4913	1	2	1.4913	1	2
	0.25	1.4913	1	2	1.4913	1	2
	0.5	1.4913	1	2	1.4913	1	2
	0.75	1.4913	1	2	1.4913	1	2
	0.95	1.4913	1	2	1.4913	1	2

Table 3

Optimal parameters of the SSMGR-AI chart for minimizing the EARL (δ_{min}, δ_{max}) values when $ARL_0 = 200$

$\delta_{\min}$	$\delta_{\max}$	ρ	$n = 5$			$n = 7$		
			k	W_1	W_2	k	W_1	W_2
0.1	0.5	0	2.1052	1	66	2.0690	1	56
		0.25	2.0878	1	61	2.0690	1	56
		0.5	2.0805	1	59	2.0442	1	50
		0.75	2.0213	1	45	1.9733	1	36
		0.95	1.7763	1	14	1.7273	1	11
0.5	1.0	0	1.6365	1	7	2.0690	1	56
		0.25	1.6365	1	7	2.0690	1	56
		0.5	1.6057	1	6	2.0442	1	50
		0.75	1.5249	1	4	1.9733	1	36
		0.95	1.3857	1	2	1.7273	1	11
1.0	1.5	0	1.4674	1	3	1.3857	1	2
		0.25	1.4674	1	3	1.3857	1	2
		0.5	1.3857	1	2	1.3857	1	2
		0.75	1.3857	1	2	1.3857	1	2
		0.95	1.3857	1	2	1.3857	1	2
1.5	2.0	0	1.6365	1	7	1.3857	1	2
		0.25	1.6365	1	7	1.3857	1	2
		0.5	1.6057	1	6	1.3857	1	2
		0.75	1.5249	1	4	1.3857	1	2
		0.95	1.3857	1	2	1.3857	1	2

Table 4
Optimal parameters of the SSMGR-AI chart for minimizing the EARL (δ_{min}, δ_{max}) values when $ARL_0 = 370$

δ_{min}	δ_{max}	ρ	$n = 5$			$n = 7$		
			k	W_1	W_2	k	W_1	W_2
0.1	0.5	0	2.2660	1	106	2.2188	1	85
		0.25	2.2660	1	106	2.2188	1	85
		0.5	2.2425	1	95	2.2060	1	80
		0.75	2.1779	1	70	2.1277	1	55
		0.95	1.9256	1	20	1.8570	1	14
0.5	1	0	1.7734	1	9	1.6975	1	6
		0.25	1.7514	1	8	1.6975	1	6
		0.5	1.7263	1	7	1.6635	1	5
		0.75	1.6217	1	4	1.5678	1	3
		0.95	1.4913	1	2	1.4913	1	2
1	1.5	0	1.5678	1	3	1.4913	1	2
		0.25	1.5678	1	3	1.4913	1	2
		0.5	1.4913	1	2	1.4913	1	2
		0.75	1.4913	1	2	1.4913	1	2
		0.95	1.4913	1	2	1.4913	1	2
1.5	2	0	1.4913	1	2	1.4913	1	2
		0.25	1.4913	1	2	1.4913	1	2
		0.5	1.4913	1	2	1.4913	1	2
		0.75	1.4913	1	2	1.4913	1	2
		0.95	1.4913	1	2	1.4913	1	2

The performance of any control chart is measured by how rapidly the chart can detect a process shift. If an in-hand chart is faster than its competing charts in spotting a process shift after setting all charts in a similar in-control performance, then the in-hand chart has a better performance than the other charts. In this study, the ARL (δ) and EARL (δ_{min}, δ_{max}) criteria are used to measure the performance of the proposed SSMGR-AI chart, and compared to that of the EWMA-AI and the RS-AI charts (see Table 5-8). Abbas et al. (2014) used the simulation approach to study for ARL performance of the EWMA-AI chart. In this study, the researchers considered the optimal EWMA-AI chart in computing the ARL and EARL values. Ng et al. (2018) developed the RS-AI chart and showed that the RS AI chart outperformed the existing Synthetic-AI chart, and the performance of the seven-region RS-AI chart was better than that of the four-region RS-AI chart. Thus, this study considered only the seven-region RS-AI chart.

It is observed from Tables 5 and 6 that when $\delta = 0.1$ and $\rho \in (0, 0.25, 0.5, 0.75)$, the EWMA-AI chart performs better than the SSMGR-AI chart and RS-AI chart but when $\rho > 0.75$, the performance of the proposed SSMGR-AI chart is significantly better compared to the EWMA-AI chart. For example, when $\delta = 0.1$, $\rho = 0.75$ and $n = 0.5$ the ARL (0.1) for EWMA-AI chart is 36.41 which is smaller than that of the SSMGR-AI and the RS-AI charts, i.e. 50.31 and 66.39, respectively , while for $\rho = 0.95$, the ARL (0.1) of the SSMGR-AI chart is 10.94, which is smaller than that of the EWAM-AI and the RS-AI chart, i.e. 13.42 and 13.40, respectively (see Table 5). As the shift increases, the performance of the proposed chart is shown to be more outstanding compared to the EWMA-AI chart and the proposed chart outperforms the RS-AI chart for all size of mean shifts and correlation coefficients. When $\delta = 0.5$, $\rho = 0.75$, $n = 5$, and $ARL_0 = 200$, the ARL (δ) value of the SSMGR-AI chart is 1.92, but the corresponding values of the EWMA-AI chart and RS-AI chart are 3.85 and 4.02, respectively (see Table 5), which are greater than that of the SSMGR-AI chart. It is also observed from Table 5 that the performance of the proposed chart is better as n increases. It is also apparent in Table 5 that the performance of the charts considered improve as ρ increases.

In Table 6, for $\delta = 1$, $\rho = 0.5$, $n = 5$, $ARL_0 = 370$, the average run length values of the SSMGR-AI, EWMA-AI and RS-AI charts are 1.19, 2.26, 1.57, respectively, i.e. ARL $(\delta)_{\text{SSMGR-AI}}$ < ARL $(\delta)_{\text{RS-AI}}$ < ARL $(\delta)_{\text{EWMA-AI}}$. A similar trend is also noticeable when n = 7 (see Table 6). On the other hand, in Tables 7 and 8, it is found that for any ($\delta_{\min}$, δ_{max}) combination, for $n = 5$ or 7, the EARL values of the SSMGR-AI chart are less than that of the EWMA-AI and RS-AI charts, except for the combination ($\delta_{\min}$, $\delta_{\max}$) = (0.1, 0.5) and $\rho \leq 0.5$. When ($\delta_{\min}$, $\delta_{\max}$) = (0.1, 0.5) and $\rho \leq 0.5$, the EWMA-AI chart shows a better performance than the other charts when ARL_0s are set as 200 and 370 (see Tables 7 and 8).

The speed in which the SSMGR-AI chart is quicker in detecting a process shift compared with the existing EWMA-AI and RS-AI charts is also shown in parentheses in Tables 5 – 8, in terms of percentages. A positive (negative) percentage for a certain chart means that the SSMGR-AI chart is quicker (slower) than the said chart in the detection of a shift. For example, in Table 5, when $ARL_0 = 200$, $n = 5$, $\delta = 0.5$ and ρ = 0.25, the ARL(0.5) values of the EWMA-AI, RS-AI and SSMGR-AI charts are 6.75, 7.68 and 3.91, respectively, where these ARL(0.5) values indicate that the SSMGR-AI chart is 72.6% and 96.4% quicker than the EWMA-AI and RS-AI charts, respectively, in detecting the shift $\delta = 0.5$. Additionally, consider Table 8, where $ARL_0 = 370$, n = 7, ($\delta_{\min}$, $\delta_{\max}$) = (0.5, 1) and $\rho = 0.5$. Here, EARL($\delta_{\min}$, $\delta_{\max}$) = 3, 3.03 and 1.5, for the EWMA-AI, RS-AI and SSMGR-AI charts, respectively, and these EARL($\delta_{\min}$, $\delta_{\max}$) values show that the SSMGR-AI chart is 100% quicker than the EWMA-AI and RS-AI charts, in detecting shifts in the interval ($\delta_{\min}$, $\delta_{\max}$) = (0.5, 1).

The findings reveal that the proposed SSMGR-AI chart generally prevails over existing competing charts to detect process mean shifts, in terms of the ARL and EARL performance criteria. However, in the case of detecting small shifts ($\delta = 0.1$) and $\rho \leq$ 0.75, the EWMA-AI chart outperforms the SSMGR-AI chart.

Table 5

ARL (δ) values for the EWMA-AI, RS-AI and SSMGR-AI charts and the percentage (in parenthesis) in which the SSMGR-AI chart is quicker (positive %) or slower (negative %) than the EWMA-AI and RS-AI charts, in detecting shifts δ, when $ARL_0 = 200$

δ	ρ	$n = 5$			$n = 7$		
		EWMA-AI	RS-AI	SSMGR-AI	EWMA-AI	RS-AI	SSMGR-AI
0.1	0	59.28 (-37.3%)	108.03 (14.2%)	94.58	48.91(-34.9%)	90.69 (20.6%)	75.18
	0.25	57.16 (-37.1%)	104.72 (15.3%)	90.81	47.11 (-34.2%)	87.38 (22.1%)	71.59
	0.5	50.31 (-35.5%)	93.20 (19.6%)	77.94	41.23 (-31.1%)	76.14 (27.3%)	59.80
	0.75	36.41 (-27.6%)	66.39 (32%)	50.31	29.44 (-20.4%)	51.64 (39.5%)	36.97
	0.95	13.42 (22.7%)	13.40 (22.3%)	10.94	10.57 (39.6%)	13.40 (77%)	7.57
0.3	0	14.70 (16.5%)	20.77 (64.6%)	12.62	11.60 (32.9%)	15.14 (73.4%)	8.73
	0.25	14.05 (19.5%)	19.55 (66.2%)	11.76	11.08 (36.3%)	14.25 (75.3%)	8.13
	0.5	12.01 (30.4%)	15.85 (72.1%)	9.21	9.44 (48%)	11.60 (81.8%)	6.38
	0.75	8.15 (58.9%)	9.67 (88.5%)	5.13	6.38 (76.2%)	7.19 (98.6%)	3.62
	0.95	2.70 (95.7%)	2.10 (52.2%)	1.38	2.09 (78.6%)	2.10 (79.5%)	1.17
0.5	0	7.07 (69.2%)	8.12 (94.3%)	4.18	5.53 (85.6%)	6.06 (103.4%)	2.98
	0.25	6.75 (72.6%)	7.68 (96.4%)	3.91	5.27 (88.2%)	5.72 (104.3%)	2.80
	0.5	5.73 (83.7%)	6.33 (102.3%)	3.12	4.47 (96.1%)	4.73 (107.5%)	2.28
	0.75	3.85 (100.5%)	4.02 (109.4%)	1.92	3.01 (100.7%)	3.09 (106%)	1.50
	0.95	1.27 (25.7%)	1.08 (7%)	1.01	1.08 (8%)	1.08 (8%)	1.00
0.7	0	4.31 (97.7%)	4.55 (108.7%)	2.18	3.37 (101.8%)	3.49 (109%)	1.67
	0.25	4.11 (98.6%)	4.32 (108.7%)	2.07	3.21 (101.9%)	3.32 (108.8%)	1.59
	0.5	3.49 (101.7%)	3.63 (109.8%)	1.73	2.72 (95.7%)	2.77 (99.3%)	1.39
	0.75	2.33 (87.9%)	2.35 (89.5%)	1.24	1.80 (63.6%)	1.80 (63.6%)	1.10
	0.95	1.01 (1%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
1	0	2.54 (92.4%)	2.58 (95.5%)	1.32	1.97 (75.9%)	1.97 (75.9%)	1.12
	0.25	2.42 (90.6%)	2.45 (92.9%)	1.27	1.87 (78.1%)	1.87 (78.1%)	1.05
	0.5	2.04 (75.9%)	2.05 (76.7%)	1.16	1.57 (57%)	1.57 (57%)	1.00
	0.75	1.36 (33.3%)	1.36 (33.3%)	1.02	1.13 (13%)	1.13 (13%)	1.00
	0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
1.5	0	1.38 (34%)	1.38 (34%)	1.03	1.14 (12.9%)	1.14 (12.9%)	1.01
	0.25	1.32 (29.4%)	1.32 (29.4%)	1.02	1.11 (11%)	1.11(11%)	1.00
	0.5	1.16 (14.9%)	1.16 (14.9%)	1.01	1.04 (4%)	1.04 (4%)	1.00
	0.75	1.01 (1%)	1.01 (1%)	1.00	1.00 (0%)	1.00 (0%)	1.00
	0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
2	0	1.05 (5%)	1.05 (5%)	1.00	1.01 (1%)	1.01 (1%)	1.00
	0.25	1.04 (4%)	1.03 (3%)	1.00	1.00 (0%)	1.00 (0%)	1.00
	0.5	1.01 (1%)	1.01 (1%)	1.00	1.00 (0%)	1.00 (0%)	1.00
	0.75	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
	0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00

Table 6

ARL (δ) values for the EWMA-AI, RS-AI and SSMGR-AI charts and the percentage (in parenthesis) in which the SSMGR-AI chart is quicker (positive %) or slower (negative %) than the EWMA-AI and RS-AI charts, in detecting shifts, when $ARL_0 = 370$

δ	ρ	$n = 5$			$n = 7$		
		EWMA-AI	RS-AI	SSMGR-AI	EWMA-AI	RS-AI	SSMGR-AI
0.1	0	76.91 (-49.7%)	173.87 (13.7%)	152.87	62.11 (-46.8%)	141.58 (21.2%)	116.82
	0.25	73.83 (-49.3%)	167.60 (15%)	145.75	59.61 (-46%)	135.57 (22.9%)	110.34
	0.5	64.07 (-47.4%)	146.18 (20%)	121.84	51.75 (-42.2%)	115.58 (29.1%)	89.56
	0.75	45.25 (-38.9%)	98.73 (33.4%)	74.02	36.11 (-30.7%)	74.10 (42.3%)	52.09
	0.95	15.76 (14%)	13.40 (-3%)	13.82	12.28 (32%)	16.34 (75.7%)	9.30
0.3	0	17.34 (7.6%)	26.57 (64.8%)	16.12	13.53 (24.8%)	18.69 (72.4%)	10.84
	0.25	16.54 (10.7%)	24.83 (66.2%)	14.94	12.90 (28.4%)	17.48 (73.9%)	10.05
	0.5	14.04 (22.3%)	19.66 (71.3%)	11.48	10.92 (41.1%)	13.93 (80%)	7.74
	0.75	9.38 (53.3%)	11.42 (86.6%)	6.12	7.27 (73.1%)	8.31 (97.9%)	4.20
	0.95	3.00 (106.9%)	2.10 (44.8%)	1.45	2.32 (91.7%)	2.32 (91.7%)	1.21
0.5	0	8.09 (64.8%)	9.47 (92.9%)	4.91	6.27 (83.9%)	6.97 (104.4%)	3.41
	0.25	7.70 (68.5%)	8.92 (95.2%)	4.57	5.96 (87.4%)	6.56 (106.3%)	3.18
	0.5	6.50 (81.6%)	7.29 (103.6%)	3.58	5.03 (97.3%)	5.36 (110.2%)	2.55
	0.75	4.32 (104.7%)	4.50 (113.3%)	2.11	3.35 (108.1%)	3.44 (113.7%)	1.61
	0.95	1.36 (33.3%)	1.08 (5.9%)	1.02	1.12 (12%)	1.12 (12%)	1.00
0.7	0	4.85 (99.6%)	3.49 (43.6%)	2.43	3.76 (107.7%)	3.89 (114.9%)	1.81
	0.25	4.62 (101.7%)	3.32 (45%)	2.29	3.58 (108.1%)	3.69 (108.1%)	1.72
	0.5	3.90 (107.4%)	2.77 (47.3%)	1.88	3.03 (107.5%)	3.09 (111.6%)	1.46
	0.75	2.59 (99.2%)	1.80 (38.5%)	1.30	1.99 (76.1%)	1.98 (75.2%)	1.13
	0.95	1.02 (2%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
1	0	2.83 (103.6%)	1.97 (41.7%)	1.39	2.18 (86.3%)	2.18 (86.3%)	1.17
	0.25	2.69 (102.3%)	1.87 (40.6%)	1.33	2.07 (81.6%)	2.06 (80.7%)	1.14
	0.5	2.26 (89.9%)	1.57 (31.9%)	1.19	1.73 (61.7%)	1.72 (60.7%)	1.07
	0.75	1.48 (43.7%)	1.13 (9.7%)	1.03	1.18 (16.8%)	1.18 (16.8%)	1.01
	0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
1.5	0	1.49 (44.7%)	1.14 (10.7%)	1.03	1.19 (17.8%)	1.19 (17.8%)	1.01
	0.25	1.42 (37.9%)	1.11 (7.8%)	1.03	1.15 (15%)	1.15 (15%)	1.00
	0.5	1.23 (21.8%)	1.04 (3%)	1.01	1.06 (6%)	1.06 (6%)	1.00
	0.75	1.02 (2%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
	0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
2	0	1.08 (8%)	1.07 (7%)	1.00	1.01 (1%)	1.01 (1%)	1.00
	0.25	1.06 (6%)	1.06 (6%)	1.00	1.01 (1.01%)	1.01(1.01%)	1.00
	0.5	1.02 (2%)	1.02 (2%)	1.00	1.00 (0%)	1.00 (0%)	1.00
	0.75	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
	0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00

Table 7

EARL (δ_{min}, δ_{max}) values for the EWMA-AI, RS-AI and SSMGR-AI charts and the percentage (in parenthesis) in which the SSMGR-AI chart is quicker (positive %) or slower (negative %) than the EWMA-AI and RS-AI charts, in detecting shifts, when $ARL_0 = 200$

			n = 5			*n* = 7		
δ_{min}	δ_{max}	ρ	EWMA-AI	RS-AI	SSMGR-AI	EWMA-AI	RS-AI	SSMGR-AI
0.1	0.5	0	20.26 (-16%)	31.44 (30.4%)	24.11	16.30 (-8.8%)	24.11(34.9%)	17.87
		0.25	19.45 (-14.8%)	29.91 (31%)	22.83	15.61 (-7.2%)	22.88 (36%)	16.82
		0.5	16.82 (-10%)	25.08 (34.3%)	18.68	13.46 (-1%)	19.03 (39.9%)	13.60
		0.75	11.74 (5.4%)	16.05 (44.1%)	11.14	9.33 (17.8%)	12.06 (52.3%)	7.92
		0.95	4.14 (66.9%)	4.45 (79.4%)	2.48	3.25 (72%)	3.35 (77.2%)	1.89
0.5	1.0	0	4.14 (84.8%)	4.50 (100.9%)	2.24	3.33 (-81.4%)	3.44 (-80.7%)	17.87
		0.25	4.25 (100.5%)	4.27 (101.4%)	2.12	3.17 (-81.2%)	3.26 (-80.6%)	16.82
		0.5	4.05 (128.8%)	3.57 (101.7%)	1.77	2.69 (-80.2%)	2.73 (-79.9%)	13.60
		0.75	3.45 (171.7%)	2.32 (82.7%)	1.27	1.80 (-77.3%)	1.79 (-77.4%)	7.92
		0.95	2.31 (131%)	1.04 (4%)	1.00	1.01 (-46.6%)	1.01 (-46.6%)	1.89
1.0	1.5	0	1.87 (65.5%)	1.87 (65.5%)	1.13	1.46 (40.4%)	1.46 (40.4%)	1.04
		0.25	1.78 (60.4%)	1.78 (60.4%)	1.11	1.40 (35.9%)	1.39 (35%)	1.03
		0.5	1.51 (43.8%)	1.51 (43.8%)	1.05	1.22 (20.8%)	1.22 (20.8%)	1.01
		0.75	1.12 (10.9%)	1.12 (10.9%)	1.01	1.03 (3%)	1.03 (3.%)	1.00
		0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
1.5	2.0	0	1.17 (15.8%)	1.17 (15.8%)	1.01	1.05 (5%)	1.05 (5%)	1.00
		0.25	1.14 (12.9%)	1.14 (12.9%)	1.01	1.00 (0%)	1.00 (0%)	1.00
		0.5	1.06 (6%)	1.06 (6%)	1.00	1.01 (1%)	1.01 (1%)	1.00
		0.75	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
		0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00

Table 8

EARL (δ_{min}, δ_{max}) values for the EWMA-AI, RS-AI and SSMGR-AI charts and the percentage (in parenthesis) in which the SSMGR-AI chart is quicker (positive %) or slower (negative %) than the EWMA-AI and RS-AI charts, in detecting shifts (δ_{min} , δ_{max}), when ARL_0=370

			n = 5			*n* = 7		
δ_{max}	δ_{max}	ρ	EWMA-AI	RS-AI	SSMGR-AI	EWMA-AI	RS-AI	SSMGR-AI
0.1	0.5	0	24.83 (-28.8%)	44.38 (27.2%)	34.89	19.73 (-21.6%)	33.00 (-21.6%)	25.17
		0.25	23.79 (-27.4%)	41.98 (28%)	32.79	18.86 (-19.9%)	31.13 (-19.9%)	23.54
		0.5	20.40 (-22.7%)	34.48 (30.7%)	26.38	16.14 (-13.4%)	25.35 (-13.4%)	18.63
		0.75	13.99 (-6.6%)	20.96 (39.9%)	14.98	10.99 (6.6%)	15.24 (6.6%)	10.31
		0.95	4.71 (64.1%)	3.68 (28.2%)	2.87	3.69 (74.9%)	3.82 (74.9%)	2.11

Table 8 *(Continued)*

			n = 5			n = 7		
$\delta_{\max}$	$\delta_{\max}$	ρ	EWMA-AI	RS-AI	SSMGR-AI	EWMA-AI	RS-AI	SSMGR-AI
0.5	1.0	0	4.79 (90.1%)	5.10 (102.4%)	2.52	3.72 (100%)	3.84 (100%)	1.86
		0.25	4.56 (92.4%)	4.82 (103.4%)	2.37	3.55 (100.6%)	3.65 (100.6%)	1.77
		0.5	3.86 (99%)	4.00 (106.2%)	1.94	3.00 (100%)	3.03 (100%)	1.50
		0.75	2.57 (93.2%)	2.58 (94%)	1.33	1.98 (72.2%)	1.97 (72.2%)	1.15
		0.95	1.06 (6%)	1.06 (6%)	1.00	1.01 (1%)	1.01 (1%)	1.00
1.0	1.5	0	2.07 (80%)	2.06 (79.1%)	1.15	1.59 (50%)	1.58 (50%)	1.06
		0.25	1.97 (74.3%)	1.96 (73.5%)	1.13	1.51 (45.2%)	1.50 (45.2%)	1.04
		0.5	1.65 (54.2%)	1.64 (53.3%)	1.07	1.30 (27.5%)	1.29 (27.5%)	1.02
		0.75	1.16 (14.9%)	1.16 (14.9%)	1.01	1.04 (4%)	1.04 (4%)	1.00
		0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
1.5	2.0	0	1.24 (22.8%)	1.23 (21.8%)	1.01	1.07 (7%)	1.07 (7%)	1.00
		0.25	1.19 (17.8%)	1.19 (17.8%)	1.01	1.05 (5%)	1.05 (5%)	1.00
		0.5	1.09 (9%)	1.09 (9%)	1.00	1.02 (2%)	1.02 (2%)	1.00
		0.75	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00
		0.95	1.00 (0%)	1.00 (0%)	1.00	1.00 (0%)	1.00 (0%)	1.00

DISCUSSION ON THE IMPLEMENTATION OF THE PROPOSED SSMGR-AI CHART

In this section, a numerical example is given for a clear understanding on how the proposed SSMGR-AI chart can be employed. For this purpose, Statistical Analysis software (SAS) is used to generate one hundred (100) in-control observations (*S, M*) from a bivariate normal distribution, i.e. $N_2\left(\mu_{S0}+\delta\sigma_S,\mu_M,\sigma_S^2,\sigma_M^2,\rho\right)$, and followed by one hundred (100) out-of-control observations (*S, M*) are generated from same underlying distribution, where $\delta = 0.5$, $\rho = 0.25$, $\mu_{S0} = \mu_M = 0$ and $\sigma_S^2 = \sigma_M^2 = 1$ are considered. Table 9 shows that among the 27 bivariate samples, the first 15 samples are grouped from 100 in-control observations and the next 12 samples are grouped from 100 out-of-control observations. Here, each sample contains 5 observations. It is assumed that the SSMGR-AI chart is optimally designed by considering $\text{ARL}_0 = 200$, $\delta = 0.5$, $n = 5$ and the corresponding optimal parameters (k, W_1, W_2) = (1.7273, 1, 11) are chosen from Table 1.

From Table 9, it is seen that for the first sample, i.e. when $i = 1$, $\hat{\mu}_{S_1}^* = 0.089$, which is computed using Equation [4]. Since $\hat{\mu}_{S_1}^* \in \left[\text{LCL,UCL}\right]$ = [–0.748, 0.748] the first sample is conforming. In a similar manner, the procedure is repeated for samples 2 – 11 (see Table 9). At sample number 12, as $\left(\hat{\mu}_{S_2}^* = 0.878\right) \notin \left[-0.748, 0.748\right]$, the sample is known as non-conforming and hence, $Y_1 = 12$ is obtained. Since $Y_1 > W_2$, the process is considered as in-control. Other non-conforming samples are detected at samples number *i* = 18, 22, 25, 26 and 27. It follows that $Y_2 = 6$, $Y_3 = 4$, $Y_4 = 3$ and $Y_5 = Y_6 = Y_1$. Figure 2 shows that the first out-of-control signal appears at sample number 27 as $Y_5 \leq 1$ and $Y_6 \leq 11$ (see Figure 2) and the control chart's statistics $\hat{\mu}^*_{S_{26}}$ and $\hat{\mu}^*_{S_{27}}$ both fall on the same side of the target mean value of the SSMGR-AI chart. Therefore, corrective actions should be taken to bring the out-of-control process back into the in-control situation. Examples of corrective actions include removal of assignable causes, such as inferior materials, operator errors and faulty parts.

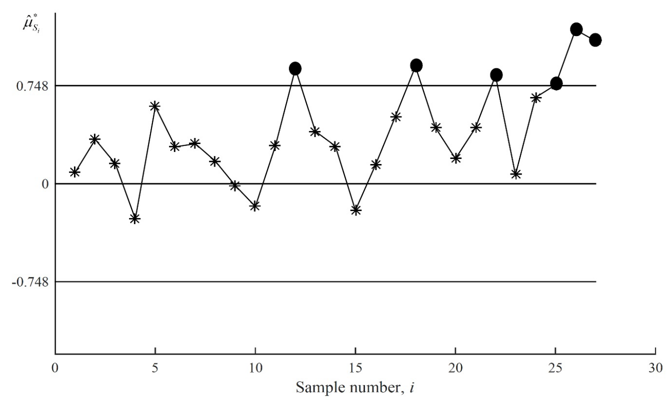

Figure 2. SSMGR-AI chart for the numerical example

Table 9

Bivariate observations (S, M) generated from normal distribution $N_2\left(\mu_{S0} + \delta\sigma_S, \mu_M, \sigma_S^2, \sigma_M^2, \rho\right)$, *where* $\mu_{S0} = \mu_M = 0$ $\sigma_S^2 = \sigma_M^2 = 1$, $\delta = 0.5$, *for the SSMG-AI chart with optimal parameters* (*k*, W_1, W_2) = (1.7273, 1, 11) *obtained from Table 1*

Sample number, *i*	S_1	S_2	S_3	S_4	S_5	M_1	M_2	M_3	M_4	M_5	$\hat{\mu}_{S_i}$	$\hat{\mu}_{M_i}$	$\hat{\mu}^*_{S_i}$
1	-0.182	0.278	1.215	-1.399	0.316	1.228	1.215	-0.834	-1.276	-1.204	0.046	-0.174	0.089
2	0.159	-0.012	1.502	-0.095	0.465	0.536	-0.042	-0.871	0.195	1.516	0.404	0.267	0.337
3	0.162	0.089	-0.698	-0.295	1.980	1.630	-0.863	0.761	0.478	-0.109	0.248	0.379	0.153
4	-0.187	0.595	-0.617	-0.832	-0.271	-0.721	1.938	-1.529	-0.025	0.464	-0.262	0.026	-0.269
5	-0.367	1.302	0.216	0.986	0.860	-0.893	-0.524	0.619	0.762	0.261	0.599	0.045	0.588

Table 9 *(Continued)*

Sample number, i	S_1	S_2	S_3	S_4	S_5	M_1	M_2	M_3	M_4	M_5	$\hat{\mu}_{S_i}$	$\hat{\mu}_{M_i}$	$\hat{\mu}^*_{S_i}$
6	0.598	1.783	-0.893	0.902	-1.681	-0.476	-0.572	-1.845	0.425	-0.338	0.142	-0.561	0.282
7	-0.867	1.071	1.074	-0.109	0.378	-0.047	1.612	-0.378	-0.011	-1.087	0.309	0.018	0.305
8	0.101	-0.621	1.941	0.949	-0.714	1.903	0.968	-1.404	1.036	0.675	0.331	0.636	0.172
9	0.336	-0.585	0.486	-0.964	0.808	1.015	0.273	0.861	-1.425	-0.105	0.016	0.124	-0.015
10	-0.308	-0.470	0.005	0.270	-0.589	-0.779	-0.915	-0.009	-0.321	1.133	-0.218	-0.178	-0.174
11	0.833	1.304	-0.879	1.194	-0.234	2.473	1.216	-0.231	0.478	-0.840	0.443	0.619	0.288
12	1.792	0.145	-0.173	1.696	1.718	-0.168	0.231	-0.305	1.993	1.414	1.036	0.633	0.878*
13	-0.003	0.733	-1.179	1.302	1.031	-0.078	0.111	-2.008	-0.449	1.959	0.377	-0.093	0.400
14	1.294	0.533	0.536	-0.246	-0.142	0.575	1.776	1.943	-0.290	-1.852	0.395	0.430	0.287
15	0.846	-0.985	-0.700	0.155	0.280	0.133	0.385	0.993	0.266	0.674	-0.081	0.490	-0.203
16	-0.810	1.187	0.254	0.567	-0.102	-0.878	-0.718	2.888	0.658	-0.410	0.219	0.308	0.142
17	0.490	1.730	-0.034	0.043	0.193	0.046	0.947	-1.152	-0.230	-0.196	0.485	-0.117	0.514
18	1.255	-1.281	2.459	0.597	1.829	-0.940	-0.591	1.388	0.350	1.327	0.972	0.307	0.895*
19	-0.587	0.559	1.491	0.624	0.745	-0.970	0.663	1.838	0.183	1.059	0.566	0.555	0.428
20	-0.599	1.040	1.017	-2.149	1.463	0.497	0.587	0.575	-1.335	-1.053	0.154	-0.146	0.191
21	1.623	-0.773	0.589	0.923	-0.025	-0.136	-0.897	-0.028	-0.334	2.224	0.467	0.166	0.426
22	0.813	0.064	1.526	1.398	-0.454	-0.197	-0.877	-1.962	1.086	-1.161	0.669	-0.622	0.825*
23	0.364	0.392	-0.746	0.893	-0.660	0.716	-0.339	-1.079	0.557	-0.264	0.049	-0.082	0.069
24	-0.260	0.710	-0.581	1.123	2.136	0.670	0.000	-1.334	-0.387	0.407	0.625	-0.129	0.658
25	1.735	-0.165	1.733	-0.556	0.238	-1.150	-0.633	-0.875	0.781	-1.261	0.597	-0.628	0.754*
26	1.545	1.094	0.335	1.738	1.462	-2.174	0.166	-0.347	1.518	2.028	1.235	0.239	1.175*
27	1.245	-0.007	0.931	2.439	0.781	-0.390	-1.631	-0.230	0.477	1.519	1.077	-0.051	1.090*

CONCLUSION

This research has proposed the SSMGR-AI chart which is based on the SSMGR charting concept of Gadre et al. (2010) to detect shifts in the process mean by using auxiliary information. Information from the study and auxiliary variables is used to derive the charting statistics of the SSMGR-AI chart, in order to monitor the mean shifts effectively. Results show that information from both these variables improves the sensitivity of the chart in detecting process mean shifts. The SSMGR-AI chart reduces to the standard SSMGR chart when $\rho = 0$. The construction procedure, optimal design, performance evaluation and implementation of the chart are elaborated in this study. Additionally, we have presented the methodology and tables of optimal parameter combinations of the SSMGR-AI chart.

The optimization algorithm developed enables the optimal SSMGR-AI chart in minimizing the ARL_1 and $EARL_1$ values, for known and unknown shift sizes, respectively.

A numerical example is also given to illustrate the construction and implementation of the proposed chart. In terms of the ARL and EARL criteria, the SSMGR-AI chart is shown to perform significantly better than the existing RS-AI chart in detecting all sizes of mean shifts, while the EWMA-AI chart performs better in detecting small shifts. However, for moderate and large shifts, the SSMGR-AI chart outperforms the EWMA-AI chart as the former has lower ARL_1 and $EARL_1$ values than the latter. Thus, the SSMGR-AI chart is deemed as an effective AI chart among existing AI charts, for monitoring the process mean.

As this study is based on the univariate SSMGR-AI chart, future research can be done on the construction of a multivariate SSMGR-AI chart for detecting shifts in the process mean vector. Furthermore, the proposed control charting concept can be extended to the monitoring of process variability or a simultaneous monitoring of the process mean and variance.

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